Capital Adequacy and Dependence

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Abstract

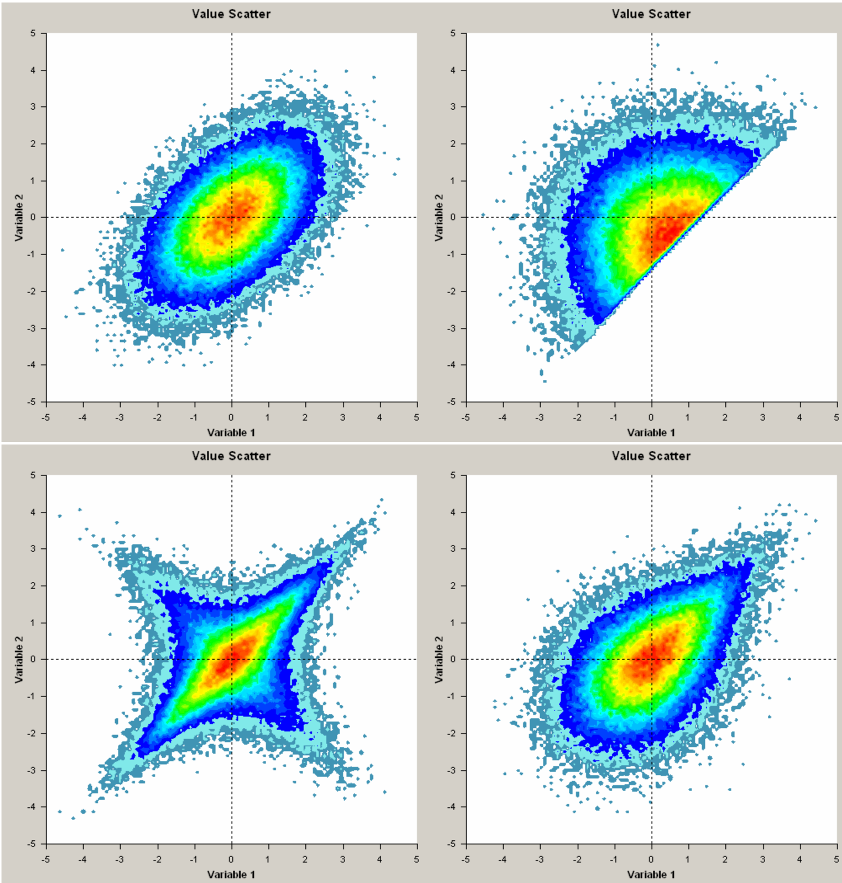
One of the key elements in any stochastic model is how the dependence between variables are modelled. A traditional way to model the dependence is to use linear correlation. This will often understate the probability of failure. Another approach to modelling dependencies is to use copulas. These allow the practitioner to introduce tail dependence into the interaction, placing greater reliability on extreme results such as those generated in capital modelling. This paper considers the impact of the choice of dependency structure on the outcome of capital modelling and describes a practical approach to modelling tail dependencies using a Gumbel dependency structure.

1. Introduction

Over the past ten years many insurers have familiarised themselves with Dynamic Financial Analysis as a means of estimating and controlling their exposure to risk concentrations. One of the key concepts underpinning the measure of concentration has been that of linear correlations.

However, correlations are constrained in describing risk concentration because they describe the concentration in only one dimension. That is, linear correlations may give an indication of the *strength* of the concentration, but do not give any indication of the *shape* of the concentration. Figure 1.1 below shows four bivariate normal distributions with linear correlation of 0.5. Despite being equally correlated normal bivariates, they are clearly very different joint distributions. In particular, the first joint distribution represents two variables that are subject to linear correlation, whilst the fourth joint distribution represents two variables that are highly "correlated" in the tail and only moderately "correlated" elsewhere.

Figure 1.1 - Dependency Structures for Equally Correlated Normal Bivariates



The fourth dependency structure shown is very common in insurance. In fact, experience between classes is often dependent in the right-hand tail for reasons such as:

- Catastrophe events affecting more than one class of business
- Superimposed inflation having an impact across a variety of long-tail classes
- Underwriting guidelines often invoke consistency across classes
- Reserving philosophies may be consistent across classes, particularly if the same actuary is reserving multiple classes
- The impact of the insurance cycle.

Copulas are mathematical tools that allow us to describe risk concentrations in terms of shape as well as strength. One such copula which exhibits right-hand tail dependence and has tractable mathematical properties is the Gumbel copula. This makes it very useful for actuaries projecting probabilities of failure where tail dependence exists. This paper attempts to provide a practical guide to using the Gumbel copula in Dynamic Financial Analysis, as well as identifying the impact of dependency structures on capital adequacy estimates.

Section 2 briefly covers the theory of correlations and dependence. The purpose of this paper is to provide a practical example of the theory in application, but some basics are required to highlight the deficiencies in using linear correlation as the only form of dependence.

Section 3 similarly comprises a brief overview of the Gumbel. It focuses on the key properties that allow the practitioner to utilise it for insurance applications, whilst also covering the potential drawbacks.

Section 4 highlights the key elements of APRA's requirements for Internal Models that require special attention when determining an appropriate dependency structure.

Section 5 gives an overview of a parameterisation process for the Gumbel that can be incorporated into the development of a DFA model. A worked example is provided.

Section 6 takes the worked example through to conclusion, highlighting the importance of the choice of dependency structure. Stress tests on the dependency structures and parameters are shown.

Section 7 summarises the conclusions reached within the body of the paper.

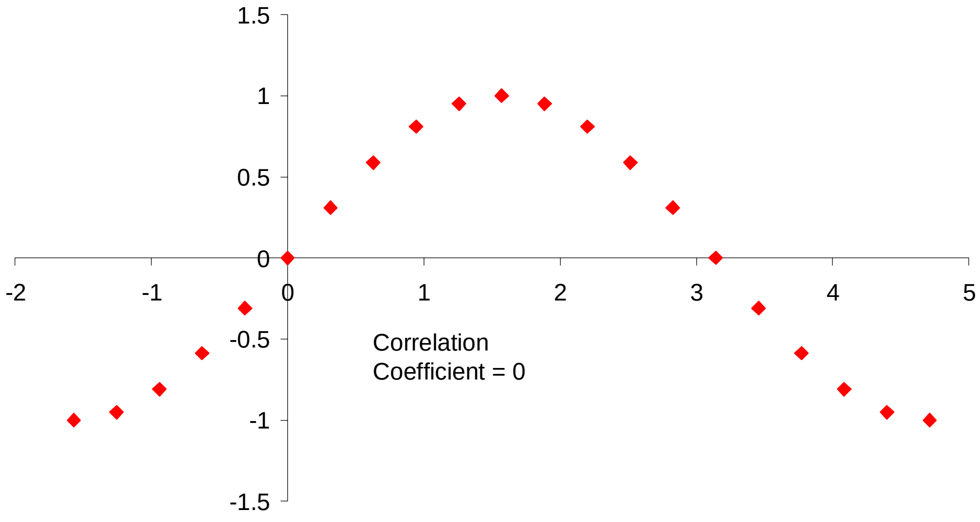
Appendices A and B give further technical discussion on choosing an appropriate dependency structure and dealing with multidimensionality, whilst Appendix C gives further details on the worked example.

2. Correlations and Dependence

Dependence is the extent to which a relationship exists between two variables. The most common form of dependence is, in fact, no dependence at all or independence. Proving that two variables are independent, however, can be extremely difficult as lack of correlation does not imply independence.

A correlation is a measure of the dependence between two variables. The type of correlation defines the aspect of dependence to be measured. Most practitioners are familiar with the concept of *linear correlation* which measures the extent to which two variables are linearly related. But variables can be dependent without being linearly related. Figure 2.1 below shows such a relationship, where $y=\sin(x)$.

Figure 2.1 - Linear Correlation for Non-Linear Relationship



By construction the two variables in Figure 2.1 are perfectly dependent, yet under the measure of linear correlation, they are uncorrelated. Because we are trying to approximate a sinusoidal relationship with a linear one, linear correlation is a poor measure of dependence for this relationship.

A similar concept to that of correlation is that of concordance. Concordance is a measure that indicates whether large values in one variate gives rise to large values in the other variate and vice versa. Unlike correlation, concordance is not a measure of the absolute levels of each variate, but rather of the rank of the variates.

Measures of Correlation and Concordance

There are many different measures of correlation and concordance. Three that are of particular use are:

- Pearson correlation
- Spearman's rho

- Kendall's tau.

Pearson Correlation (ρ)

Pearson correlation is also known as linear correlation, normal correlation or more commonly, simply correlation. It is a measure of the linear relationship between two or more variates. Pearson correlation is given by:

$$\rho = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$$

Spearman's Rho (ρ_s)

Spearman's rho is also known as rank correlation. Technically speaking it is a measure of concordance rather than correlation, but it is actually the Pearson correlation of the rank of the variates, hence the name rank correlation. It can be estimated from an underlying data set by:

$$\rho_s = \frac{n\sum_i X_i Y_i - \left(\sum_i X_i\right)\left(\sum_i Y_i\right)}{\sqrt{\left[n\sum_i X_i^2 - \left(\sum_i X_i\right)^2\right]\left[n\sum_i Y_i^2 - \left(\sum_i Y_i\right)^2\right]}}$$

where the X_i's and Y_i's represent the rank of the variates. Rank correlation is often a much better measure than Pearson correlation since it does not rely on linearity in the relationship.

Kendall's Tau (τ)

Kendall's tau is a measure of the concordance of the bivariate distribution. Kendall's tau can be estimated from an underlying data set by:

$$\tau_n = \frac{\sum_{i<j} \text{sign}\left[(X_{1i} - X_{1j})(X_{2i} - X_{2j})\right]}{\binom{n}{2}}$$

This summation is across all possible pairs of observations.

This is a measure of the concordance of the data set, which, in layman's terms, is the likelihood of large values in one variable being paired with large values in the other variable, and vice-versa.

3. A Brief Introduction to Copulas

This section is not intended to be a theoretical guide to copulas. Rather the intention is to give the reader enough of a taste to understand why actuaries may wish to consider copulas as an integral part of modelling capital adequacy. For a detailed discussion on the theory of copulas I would recommend Mari and Kotz [2001], Nelsen[1999] and Joe [1997] as treatises on the topic.

What is a Copula?

Copulas are mathematical mappings that describe how two or more variates are related. For actuaries, any form of stochastic modelling inevitably involves choosing and sampling from copulas. Most actuaries are likely to be blissfully unaware of their implicit choice of copula for a variety of stochastic modelling work.

There are two very common copulas that most actuaries will have dealt with through past experience; most likely without realising that they are indeed copulas. These are:

- The independent copula – Simply indicating that two variates are independent
- The normal copula – Commonly referred to as "correlation", this copula indicates two or more variates that are linked via a normally distributed error function, with the correlation coefficient indicating how close to a linear relationship there is between the two variables

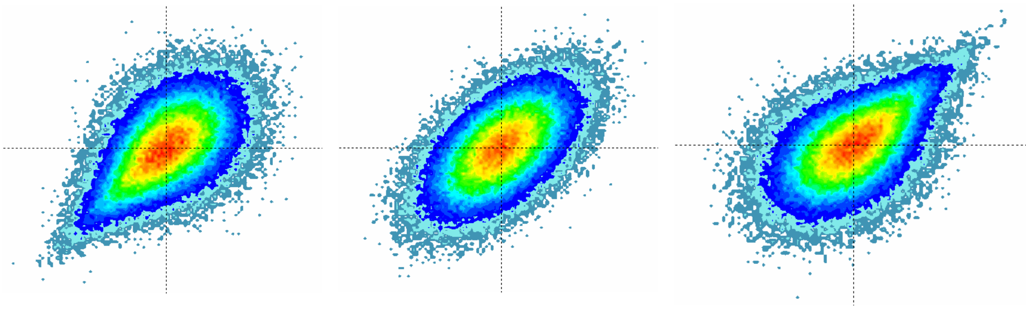
However, simply describing how strong a relationship exists between the variates is often not enough to describe the dependence, in much the same way that a mean and standard deviation may not adequately describe a distribution. Copulas allow practitioners to describe both the strength of the relationship and the shape of the relationship.

Copulas allow practitioners to choose whether a relationship is strongest in the tail or in the body of the distributions. They can determine whether the relationship is:

- Symmetrical – For example, exceptionally good results in Liability are more likely if there are exceptionally good results in CTP and poor results in Liability are more likely if there are poor results in CTP
- Left Tailed – Exceptionally good results in Liability are more likely if there are exceptionally good results in CTP, but poor results in Liability do not necessarily mean poor results in CTP nor vice versa
- Right Tailed – Exceptionally good results in Liability do not necessarily mean exceptionally good results in CTP, nor vice versa, but poor results in Liability are more likely if there are poor results in CTP.

This idea is illustrated in Figure 3.1 below.

Figure 3.1 – Left Tailed, Symmetrical and Right Tailed Copulas

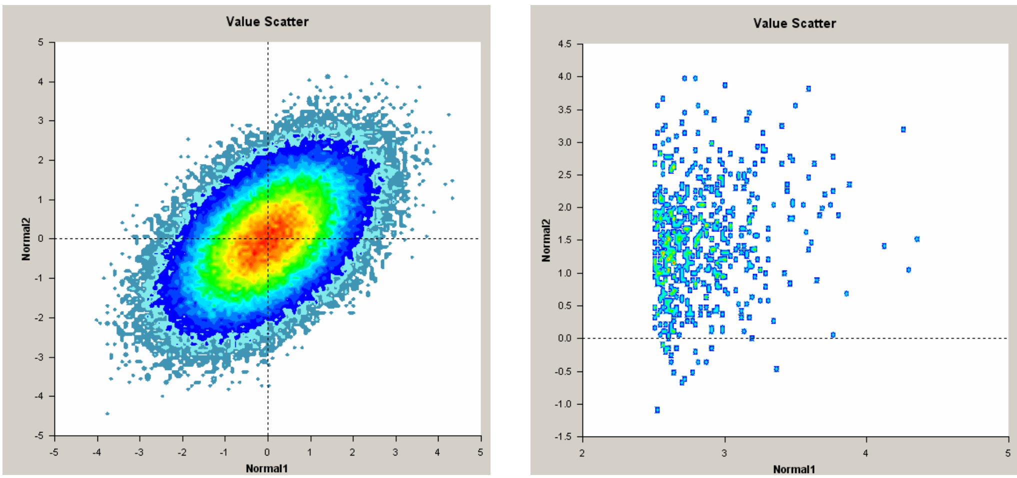


Choosing to model relationships as linear correlations implies the relationship is symmetrical and is strongest close to the means of the individual distributions. At times this can be inappropriate.

Extreme Value Copulas

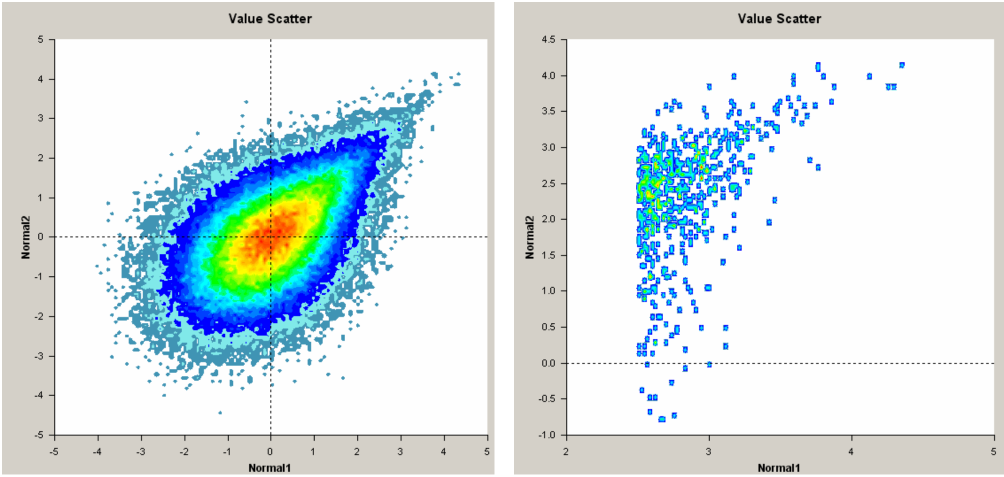
One of the major shortcomings of using linear correlations to describe extreme probabilities is that the correlation weakens as results become more extreme. Figure 3.2 shows two normal distributions with linear rank correlation coefficient of 0.5. When we zoom in on the right hand tail, the rank correlation in the tail is only 0.15.

Figure 3.2 - Reducing Correlations in the Tail



Analogous to the idea of using a fat-tailed distribution such as the Pareto to model large claims, extreme value copulas maintain the dependency structure in the tail of the joint distribution. Figure 3.3 below shows two normal distributions joined using an extreme value copula. Again, the rank correlation is 0.5. This time when we zoom in on the right hand tail, the dependency structure is maintained, with the rank correlation in the tail being 0.46.

Figure 3.3 - Extreme Value Copulas Maintain the Relationships in the Tail



Allowing practitioners the freedom to model effective risk concentrations in the tail of the joint distribution is a major advantage for accurately estimating tail probabilities.

Archimedean Copulas

There are two additional properties that are crucial to using copulas in practice. These are:

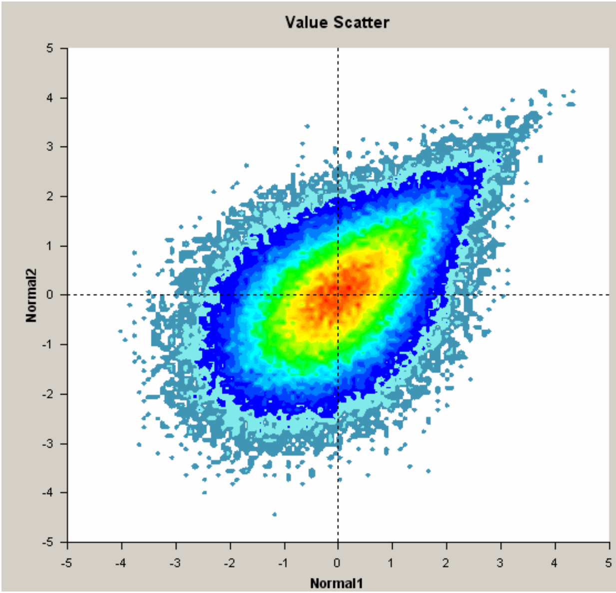
- There must be an easy way to sample from the copula
- There must be an obvious way to extend the copula to multiple dimensions.

One important family of copulas that exhibits both properties is referred to as Archimedean copulas. Within the Archimedean family of copulas, one particular copula is also an extreme value copula. This copula is known as the Gumbel-Hougaard copula. It is commonly referred to as the Gumbel copula, although Gumbel's name is attached to a number of other copulas as well.

The Gumbel Copula

The Gumbel copula is a right tailed extreme value copula that can be used to model tail dependencies in insurance. Rather than the symmetrical nature of normal correlations, the Gumbel copula exhibits a "comet-like" shape which is indicative of its right tailed extreme value nature. Figure 3.4 below shows a Gumbel copula applied to two normal distributions.

Figure 3.4 - The Gumbel Copula



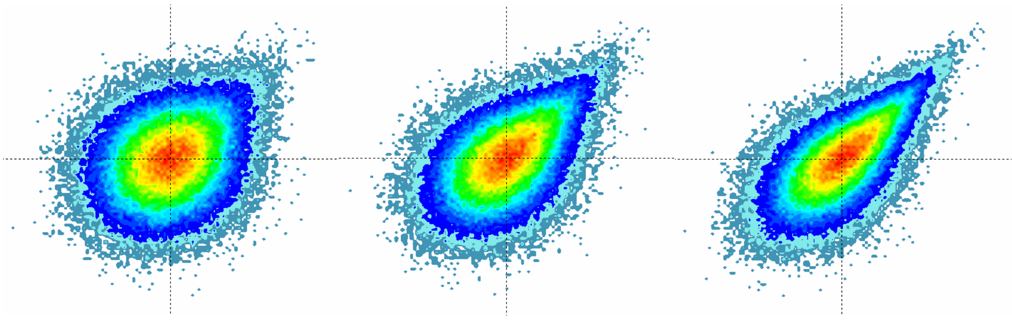
For modelling capital adequacy the Gumbel has some very useful properties, but also has a few drawbacks. The obvious advantage is that the right tailed extreme value nature of the Gumbel allows actuaries to model a range of tail relationships that would otherwise be unavailable. The drawbacks are:

- Due to the mechanism for extending to multiple dimensions, applying a Gumbel structure to n classes of business involves specifying only (n-1) parameters. This compares to defining $n(n-1)/2$ parameters for normal correlation. This introduces some limitations to the parameterisation process
- The Gumbel does not handle negative dependence. However, other Archimedean copulas such as Frank's copula can be used instead.

Nevertheless, these drawbacks can be overcome through sensible choices of the dependency structure and parameters. See Appendix B for further details.

The Gumbel is parameterised in terms of a tail parameter, alpha. This indicates the strength of the relationship in the tail. Alpha must be greater than or equal to 1, with alpha equal to one indicating independence. As alpha increases the Gumbel dependency structure becomes more "comet-like". Parameters are likely to fall in the range of 1 to 2. Figure 3.5 below shows the relationship as alpha increases.

Figure 3.5 – Tail Dependence for Alphas of 1.2, 1.5 and 1.9



4. Dependence and APRA Internal Models

One of the primary uses of Dynamic Financial Analysis is the determination of an appropriate level of capital to support the desired level of risk for an insurer. Insurers are becoming increasingly aware that any prescriptive method for determining capital adequacy may not necessarily accurately reflect the riskiness of the business.

The Australian Prudential Regulation Authority ('APRA') appear to be leading the way in terms of general insurance regulators worldwide in acknowledging that a "square peg, round hole" approach of determining capital by a series of pre-defined loadings may not be appropriate for all players in the market. The opportunity for Australian insurers to present their portfolios as efficient users of capital through the advent of the Internal Model places Australia at the forefront of regulatory thinking.

Whilst the rationale for the adoption of the concept of Internal Models is sound, the reality of their implementation is one of a process of enlightenment for both the insurer and APRA. In order for APRA to approve the validity of any Internal Model, the insurer must demonstrate that they have adequate controls over the veracity of the model and that it is a realistic model of the underlying riskiness of the business.

In particular, the insurer must demonstrate a very clear understanding of the riskiness of the tail and events that contribute heavily to the likelihood of failure. To do this properly, an insurer must be able to describe the impact of risk accumulation and risk concentration in the tail. One of the key elements is therefore how the insurer models the dependencies between sources of risk. Often a simple correlation matrix will understate the probability of failure.

Quantitative Requirements of an Internal Model

APRA Guidance Note GGN 110.2 specifies a range of quantitative requirements that must be met in any Internal Model. The Guidance Note broadly classifies these into four categories:

- Investment Risk
- Insurance Risk
- Operational Risk
- Correlation between Risk Classes

The Guidance Note does indicate that these categories are not necessarily an exhaustive source of the risk to which an insurance company is exposed.

Dependencies are an important element of the correlation between risk classes. GGN 110.2 states that "*an insurer's capital measurement model must… evaluate the interrelationships between… risks.*" Whilst the Guidance Note describes this interrelationship as a *correlation*, it does so only in a loose sense of the word. It does not necessarily imply that the interrelationships should be modelled as *linear* *correlations*. Evaluating the interrelationship between risks comprises of two key elements:

- Determining the *strength* of the interrelationship, and
- Determining the *shape* of the interrelationship.

This means that dependency structures play an integral part in any model likely to be put forward as an Internal Model. As is shown in subsequent sections, the choice of dependency structure can make a significant difference to the estimated probability of failure of the insurer.

Guidance Note GGN 110.2 also discusses the concept of stress testing any Internal Model. This involves stress testing the key elements of the model and key parameters. The subsequent sections also discuss the uncertainty in both the dependency structure chosen and the dependency parameters.

5. A Practical Guide to Fitting a Gumbel

Although the theory of copulas is not new, its application to stochastic modelling in insurance is still in its infancy. The need to adequately model tail dependency in insurance is an obvious one, meaning that extreme value copulas are of particular interest to actuaries. As discussed earlier, because of its mathematically tractable nature the Gumbel copula has attracted much attention.

What follows is a description of an approach that can be used to determine a suitable tail dependence structure, based on the use of the Gumbel copula. This is supplemented by a worked example based on eight classes of business.

It should be noted that the Gumbel is one of many dependency structures that can be implemented by the DFA practitioner. Further discussion on choosing an appropriate dependency structure can be found in Appendix A.

The key steps involved in determining an appropriate parameterisation of a Gumbel dependency structure are as follows:

- Assess pair-wise best fits
- Overcome issues with multi-dimensionality
- Determine an appropriate relational structure.

When estimating linear correlations, the process involves only the first of these three steps. Figure 5.1 below shows a partly completed pair-wise best fit parameterisation for a linear correlations dependency structure.

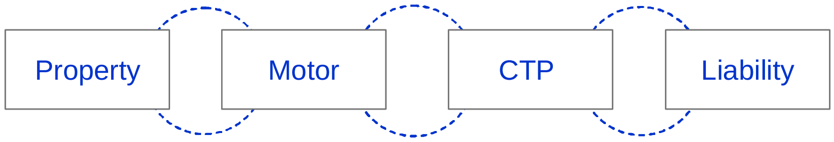
Figure 5.1 - Partial Parameterisation for Linear Correlation Matrix

	Liability	CTP	Motor	Property	…
Liability	100%	40%			…
CTP	40%	100%	25%		…
Motor		25%	100%		…
Property				100%	…
⋮	⋮	⋮	⋮	⋮	⋱

For linear correlations the actuary can select $n(n-1)/2$ correlation coefficients to describe the dependence between the n classes of business. When using a Gumbel, the practitioner can only select (n-1) parameters to describe the dependence between the n classes of business, with the remaining pair-wise relationships determined implicitly. Whilst this may seem like a substantial disadvantage relative to linear correlations, the practitioner has the choice of which relationships they explicitly define. This is done through the relational structure.

The relational structure is a series of links between two or more classes of business. The relationship between classes not explicitly linked is determined implicitly by the intermediate links between the two classes. Figure 5.2 below shows an example of one such relational structure.

Figure 5.2 - Typical Relational Structure



In the example above, the relationship between Property and Motor is explicit, whilst the relationship between Motor and Liability is implicit, since they are not explicitly linked. That means that the practitioner can explicitly choose the level of tail dependence between Property and Motor, but the tail dependence between Motor and Liability is determined by the selections between Motor and CTP and then CTP and Liability. Nevertheless, the practitioner has freedom to choose a relational structure that is sensible. This means that the relational structure should be chosen so that the most important pair-wise relationships can be explicitly modelled.

Because the practitioner is allowed only to choose (n-1) parameters to describe $n(n-1)/2$ pair-wise relationships there are sometimes problems with multidimensionality. These are largely theoretical in nature and are discussed further in Appendix B.

Details of the Worked Example

Throughout the remaining sections the discussion is supplemented by a worked example showing the determination of a Gumbel-based dependency structure and the impact of tail dependence on levels of capital. The worked example consists of a fictional start-up general insurance company being modelled for a single year and projects the insurer's solvency at year end. The insurer writes the following classes of business:

- Liability
- Workers' Compensation
- Compulsory Third Party
- Professional Indemnity
- Commercial Property
- Commercial Motor
- Home & Contents
- Domestic Motor.

Property catastrophes are modelled as an explicit dependence and impact Commercial Property, Commercial Motor, Home & Contents and Domestic Motor. Losses from catastrophes are not allocated back to the individual classes. For simplicity the insurer has excess of loss coverage for large claims set at a very high retention, so recoveries can be ignored. Other than catastrophe events, losses are being modelled using a lognormal distribution for the loss ratio. The dependency structure will apply to non-catastrophe loss ratios simulated for the year. Further details of the worked example can be found in Appendix C.

In practice it may also be necessary to incorporate large claims, the run-off of existing business and the insurer's reinsurance programme into the determination of an appropriate dependency structure, however the general approach remains the same.

Pair-Wise Best Fits

Whilst we ultimately wish to construct a multi-dimensional dependency structure, to ensure that the structure adequately represents the pair-wise relationships we first estimate the best fit on a pair-wise basis.

It is useful to consider both a statistical and judgemental approach to determining the pair-wise estimates. There are a number of statistical tests that can assist in determining pair-wise estimates. The statistical tests for the worked example below are based on historical APRA data from 1993 to 2001.

Kendall's Tau

From Kendall's tau, it is possible to estimate the Gumbel which gives rise to the same level of concordance. The best estimate Gumbel parameter is given by:

$$\alpha = \frac{1}{1-\tau}.$$

Table 5.1 below shows the best estimate pair-wise Gumbel parameters as measured by Kendall's tau based on the industry experience over nine years for the classes of business described in the worked example.

Table 5.1 - Best Estimate Pair-Wise Gumbel Parameters – Kendall's Tau

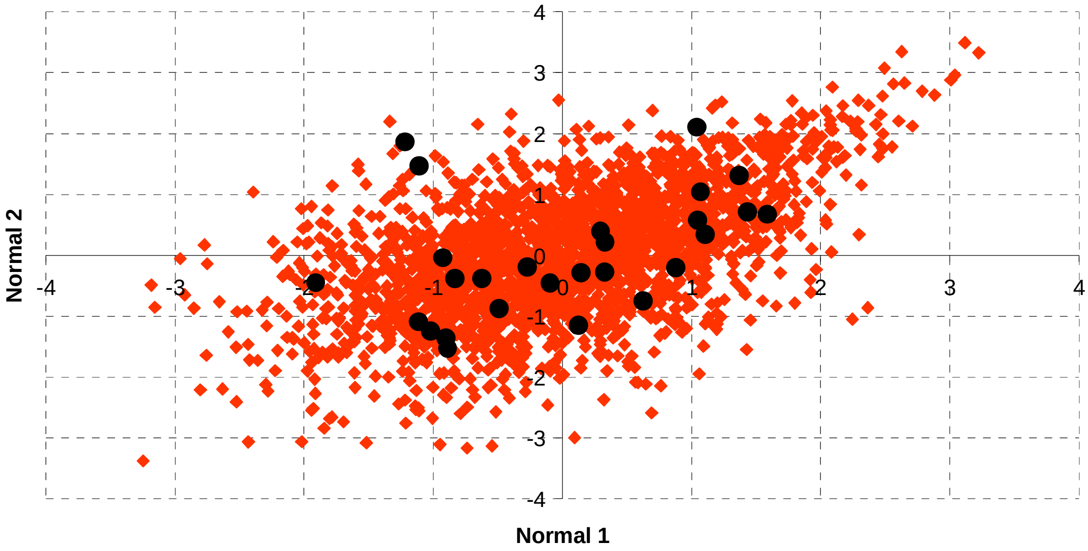
	Liability	Workers Comp	CTP	Prof Indemnity	Comm Property	Comm Motor	Domestic Property
Liability							
Workers Comp	1.20						
CTP	0.78	1.00					
Prof Indemnity	1.80	1.06	0.58				
Comm Property	1.80	1.20	0.67	1.50			
Comm Motor	1.06	1.00	1.12	1.80	0.95		
Dom Property	0.72	0.90	1.00	0.86	0.86	0.82	
Dom Motor	0.95	1.00	2.25	0.67	0.86	1.00	1.13

Note that this statistical test can give an implied alpha of less than 1, whilst the requirements for alpha must be greater than or equal to 1.

Chi-Squared Test

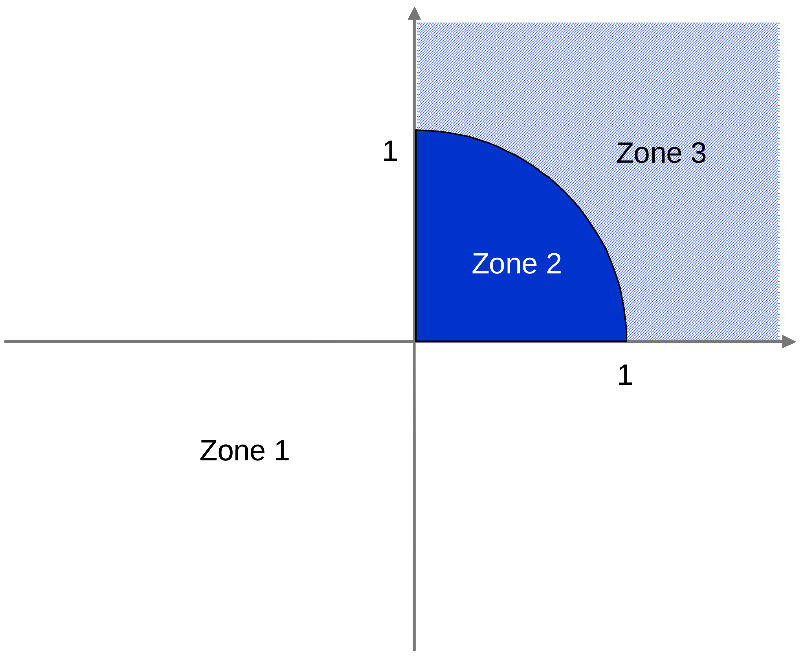
With a suitable amount of data, the practitioner may be able to map each pair of observations against simulated distributions with Gumbels of varying parameters.

Figure 5.3 - XY Plots of Paired Observations Against Gumbel with Parameter 1.6



By defining zones within the two way plot, the practitioner can determine an actual versus expected number of observations within each zone. As the number of observations increases, the practitioner can become more selective in how they define their zones. Because the worked example has only 9 historical data points, we have specified three zones. Strictly speaking, there are not enough data points for the worked example to ensure statistical significance for the chi-squared test, but the process remains the same. Figure 5.4 below illustrates the zones that we are testing.

Figure 5.4 - Zones Defined for the Worked Example



From the actual versus expected number of observations within each zone, a chi-squared statistic can be calculated. The Gumbel parameter can be estimated based on mapping with the lowest chi-squared statistic. Table 5.2 below shows the chi-squared statistic for each pair of classes for the worked example relative to a range of Gumbel parameters.

Table 5.2 - Chi-Squared Estimates for Each Class

Class 1	Class 2	Gumbel Dependency					
		1.1	1.2	1.4	1.6	1.8	2.0
Liability	Workers Comp	2.651	2.187	1.896	**1.735**	1.781	1.882
Liability	CTP	**1.764**	1.972	2.390	2.646	2.821	3.087
Liability	Prof Indemnity	2.651	2.187	1.896	**1.735**	1.781	1.882
Liability	Comm Property	2.651	2.187	1.896	**1.735**	1.781	1.882
Liability	Comm Motor	**0.171**	0.275	0.491	0.644	0.752	0.926
Liability	Dom Property	**1.412**	1.526	2.029	2.282	2.515	2.852
Liability	Dom Motor	**1.412**	1.526	2.029	2.282	2.515	2.852
Workers Comp	CTP	**0.171**	0.275	0.491	0.644	0.752	0.926
Workers Comp	Prof Indemnity	1.311	1.475	1.114	1.112	0.968	**0.825**
Workers Comp	Comm Property	0.256	0.137	0.032	**0.019**	0.048	0.119
Workers Comp	Comm Motor	1.699	1.646	0.982	0.823	0.607	**0.372**
Workers Comp	Dom Property	5.816	6.203	5.142	5.015	4.543	**3.999**
Workers Comp	Dom Motor	**0.171**	0.275	0.491	0.644	0.752	0.926
CTP	Prof Indemnity	**3.309**	3.531	4.255	4.620	4.928	5.368
CTP	Comm Property	**3.309**	3.531	4.255	4.620	4.928	5.368
CTP	Comm Motor	1.699	1.646	0.982	0.823	0.607	**0.372**
CTP	Dom Property	**0.171**	0.275	0.491	0.644	0.752	0.926
CTP	Dom Motor	0.256	0.137	0.032	**0.019**	0.048	0.119
Prof Indemnity	Comm Property	5.786	4.855	3.989	3.527	3.461	**3.428**
Prof Indemnity	Comm Motor	**0.171**	0.275	0.491	0.644	0.752	0.926
Prof Indemnity	Dom Property	0.256	0.137	0.032	**0.019**	0.048	0.119
Prof Indemnity	Dom Motor	**0.171**	0.275	0.491	0.644	0.752	0.926
Comm Property	Comm Motor	1.193	**1.078**	1.243	1.320	1.466	1.690
Comm Property	Dom Property	**0.171**	0.275	0.491	0.644	0.752	0.926
Comm Property	Dom Motor	**1.412**	1.526	2.029	2.282	2.515	2.852
Comm Motor	Dom Property	4.831	5.126	3.898	3.686	3.162	**2.548**
Comm Motor	Dom Motor	1.311	1.475	1.114	1.112	0.968	**0.825**
Dom Property	Dom Motor	1.311	1.475	1.114	1.112	0.968	**0.825**

The observations highlighted in the table show the best chi-squared test. It is also useful to note whether this is a global or a local minimum. For example, the relationship between Domestic Property and Domestic Motor may actually have a global minimum with a Gumbel parameter below 1.1.

The corresponding pair-wise best estimate of the Gumbel parameters is shown in Table 5.3 below.

Table 5.3 - Best Estimate Pair-Wise Gumbel Parameters – Chi Squared Statistics

	Liability	Workers Comp	CTP	Prof Indemnity	Comm Property	Comm Motor	Dom Property
Liability							
Workers Comp	1.6						
CTP	1.1	1.1					
Prof Indemnity	1.6	2.0	1.1				
Comm Property	1.6	1.6	1.1	2.0			
Comm Motor	1.1	2.0	2.0	1.1	1.2		
Dom Property	1.1	2.0	1.1	1.6	1.1	2.0	
Dom Motor	1.1	1.1	1.6	1.1	1.1	2.0	2.0

A Judgemental Approach

However, more often actuaries do not have access to enough representative history to rely entirely on statistical tests to determine appropriate pair-wise parameters. Therefore, much of the parameter selection will rely on judgement. Table 5.4 below shows a pair-wise estimate of the strength of the tail dependence for the worked example. The tail dependence between the property classes is Low to Medium because catastrophe events are modelled explicitly.

Table 5.4 - A Priori Estimate of Tail Dependence

	Liability	Workers Comp	CTP	Prof Indemnity	Comm Property	Comm Motor	Dom Property
Liability							
Workers Comp	Med-High						
CTP	Med-High	Med-High					
Prof Indemnity	High	Med-High	Med				
Comm Property	Low-Med	Low	Low	Low			
Comm Motor	Low	Low	Low-Med	Low	Low-Med		
Dom Property	Low	Low	Low	Low	Low-Med	Low-Med	
Dom Motor	Low	Low	Low-Med	Low	Low-Med	Low-Med	Low-Med

Whilst there is not a great deal of industry research specifically available to determine appropriate tail dependencies, the evolution of tail dependencies in insurance has emerged via correlations. Particularly since the emergence of APRA's requirements for technical liabilities to be valued at a 75th percentile of sufficiency there have been a number of papers submitted that estimate correlations between classes of business, including papers by Bateup and Reed and by Collings and White.

Whilst it is important to recognise that the correlation estimates produced in these papers are designed to estimate the relationships between classes at the 75th percentile rather than at a 99th percentile, the principles remain the same. Whilst the absolute magnitude of the relationship in the tail may not necessarily be the same as at the 75th percentile of sufficiency, whether the relationship between classes is "strong" or "weak" in the tail should follow similar drivers. Of course, there are exceptions to this rule, including:

- Any drivers that are specific to the tail, in particular catastrophe events
- Classes such as excess liability and reinsurance classes.

Both industry studies give reasonable *a priori* estimates to the level of dependence in the tail, through the assessment of correlation at the 75^{th} percentile of sufficiency of reserves. This is done by solving for the pair-wise Gumbel parameter which gives rise to an equivalent rank correlation to that implied by the industry studies.

An Overall Assessment

In practice, the approach taken is often to use the statistical tests to support the judgemental selections. Hopefully for a majority of pair-wise relationships the statistical tests do just that. For those that don't, the role of the practitioner is inevitably made decidedly harder.

Table 5.5 below shows the implied pair-wise Gumbel parameters estimated three ways for NSW CTP against all other classes.

Table 5.5 - Gumbel Pair-wise Selections for CTP

	Kendall's Tau	Chi Squared	A Priori	Selected
Liability	0.78	1.10	Med-High	1.30
Workers Comp	1.00	1.10	Med-High	1.25
Prof Indemnity	0.58	1.10	Med	1.25
Comm Property	0.67	1.10	Low	1.025
Comm Motor	1.12	2.00	Low-Med	1.15
Dom Property	1.00	1.10	Low	1.025
Dom Motor	2.25	1.60	Low-Med	1.15

Clearly there are some significant differences between the *a priori* estimates and the implied statistical tests. Three justifications for choosing parameters closer to the *a priori* estimates are:

- The history underpinning the statistical estimates may not be an appropriate benchmark. There has been significant movement in premiums, both increases and decreases, for CTP over the period. Inevitably this has an impact on loss ratios that underpin the statistical tests. Ideally, the loss ratios would be net of the underwriting cycle, but in practice this is difficult to eliminate
- There is not enough history for complete confidence in the statistical estimates
- Industry consensus would argue against CTP being essentially independent of other long tailed classes.

Table 5.6 below shows the selected pair-wise Gumbel parameters.

Table 5.6 - Pair-wise Selection of Gumbel Parameters

	Liability	Workers Comp	CTP	Prof Indemnity	Comm Property	Comm Motor	Dom Property
Liability							
Workers Comp	1.50						
CTP	1.30	1.25					
Prof Indemnity	1.50	1.50	1.25				
Comm Property	1.025	1.025	1.025	1.025			
Comm Motor	1.025	1.025	1.15	1.025	1.05		
Dom Property	1.025	1.025	1.025	1.025	1.10	1.05	
Dom Motor	1.025	1.025	1.15	1.025	1.05	1.15	1.10

The implied rank correlations for these pair-wise selections are shown in Table 5.7 below.

Table 5.7 - Implied Rank Correlations for Pair-Wise Selections

	Liability	Workers Comp	CTP	Prof Indemnity	Comm Property	Comm Motor	Dom Property
Liability							
Workers Comp	48%						
CTP	34%	29%					
Prof Indemnity	48%	34%	29%				
Comm Property	4%	4%	4%	4%			
Comm Motor	4%	4%	19%	4%	7%		
Dom Property	4%	4%	4%	4%	13%	7%	
Dom Motor	4%	4%	19%	4%	7%	19%	13%

The Relational Structure

Because not every pair-wise relationship can be uniquely constructed using a Gumbel, the practitioner has freedom to choose the relational structure that they desire between classes. The challenge is to select an appropriate relational structure to adequately describe the relationship as a whole, whilst minimising any limitations due to the lack of degrees of freedom in parameter selection. A number of considerations are relevant, including:

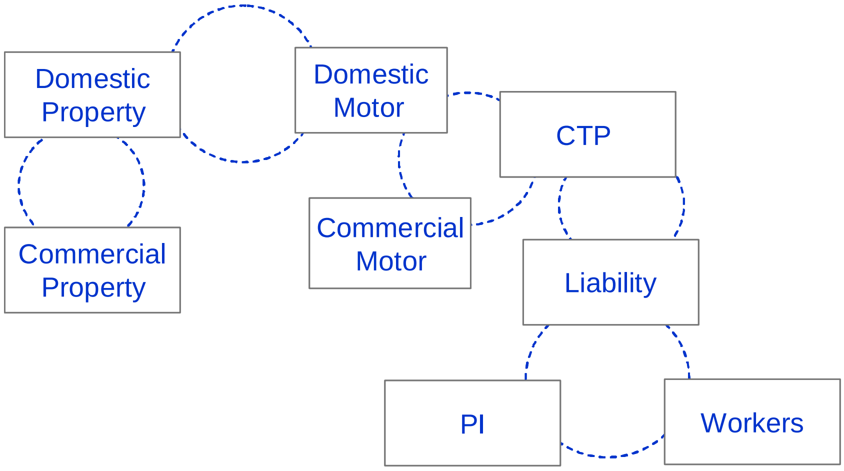
- Which classes have the strongest pair-wise relationships?
- What are the largest classes for the insurer?
- Is there a reasonable justification for “linking” two classes?

Clearly it makes sense to ensure the freedom in parameter selection is utilised to describe the strongest relationships for the largest and most significant classes where there is a real justification for tail dependence.

This means that the practitioner gets to choose some of the relationships explicitly, but by dint of their selection, must accept the implicit relationships for other pairings.

Figure 5.5 below shows one such possible relational structure for the worked example. The practitioner would assign a Gumbel parameter within each of the links.

Figure 5.5 - Potential Relational Structure for Worked Example



This relational structure links:

- Liability, Professional Indemnity and Workers' Compensation together
- Liability with CTP
- CTP with Commercial Motor and Domestic Motor
- Domestic Motor with Domestic Property
- Commercial Property with Domestic Property.

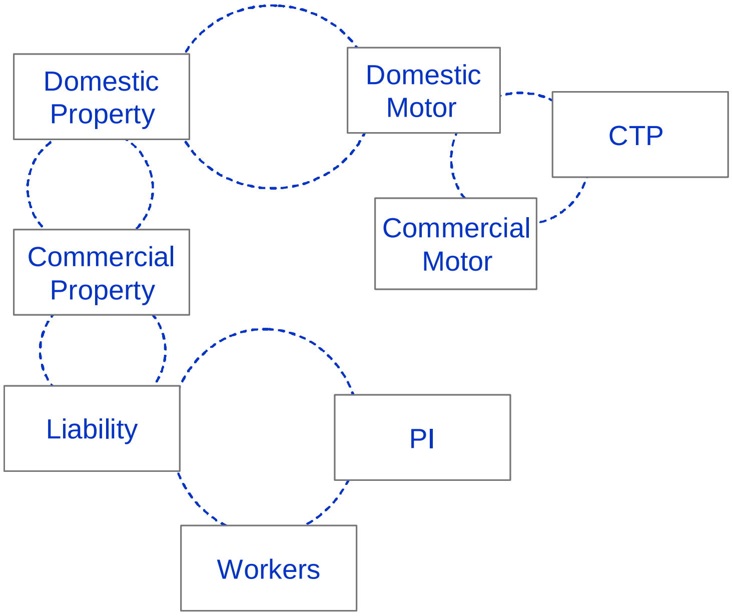
The justification for such a relational structure could be as follows:

- Jurisdictional pressures and systemic reserving uncertainty can cause the long tailed classes to move together
- CTP claims experience may be linked to Motor claims experience through a frequency effect
- The short tailed classes are likely to exhibit some small degree of tail dependence, even with catastrophes modelled explicitly.

Each pair-wise relationship weakens the more "links" between them. Therefore, whilst the relationship between CTP and Professional Indemnity may be reasonably strong, it won't be as strong as that between Liability and Professional Indemnity which sit within the same link. Nevertheless, it will produce a stronger pair-wise relationship to that between Commercial Property and Professional Indemnity, which has five links in between.

However, the choice of relational structure is not unique. An alternate relational structure is proposed in Figure 5.6 below.

Figure 5.6 - Alternate Relational Structure for Worked Example



The alternate relational structure links:

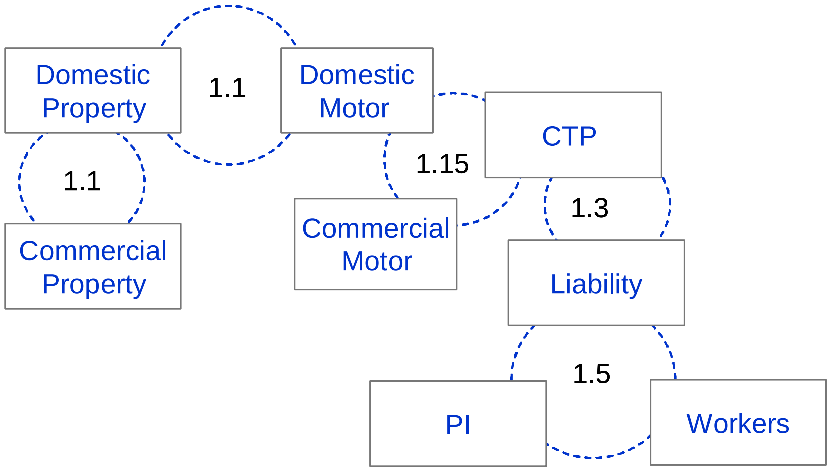
- Liability, Professional Indemnity and Workers' Compensation together
- Liability with Commercial Property
- Commercial Property with Domestic Property
- Domestic Motor with Domestic Property
- CTP with Commercial Motor and Domestic Motor.

The justification for linking liability and commercial property may arise from the theory that very large liability claims can give rise to a series of business interruption claims from a commercial property portfolio. Here the link between liability and CTP is necessarily broken.

There are a variety of other potential structures existing as well and it is important to recognise there is no one right structure to choose. However, considering the relative strength, size and justifiability of the links can narrow the choices. Nevertheless, the impact of choice of relational structure should be tested as part of any model parameterisation, much as would occur for any other key assumption.

For the worked example, the relative premium volumes suggest that the CTP and liability relationship is important, hence the first relational structure is fine. With parameters based on the pair-wise estimates between the key elements of the links, this gives a structure as shown in Figure 5.7 below.

Figure 5.7 - Relational Structure with Gumbel Parameters



The structure should be tested against the best estimate implied pair-wise rank correlations to understand the impact of the partial exchangeability problem (see Appendix B for further details). Table 5.8 below shows the implied rank correlations for each pair-wise combination, relative to the pair-wise best estimate.

Table 5.8 - Implied Rank Correlations for Dependency Structure

	Liability	Workers Comp	CTP	Prof Indemnity	Comm Property	Comm Motor	Dom Property
Liability							
Workers Comp	48%						
CTP	34%	17%					
Prof Indemnity	48%	48%	18%				
Comm Property	0%	0%	1%	1%			
Comm Motor	7%	4%	19%	4%	0%		
Dom Property	1%	1%	3%	1%	13%	3%	
Dom Motor	8%	4%	19%	4%	2%	19%	13%

Even though there were constraints in how many parameters were available to estimate, the pair-wise relationships are essentially maintained.

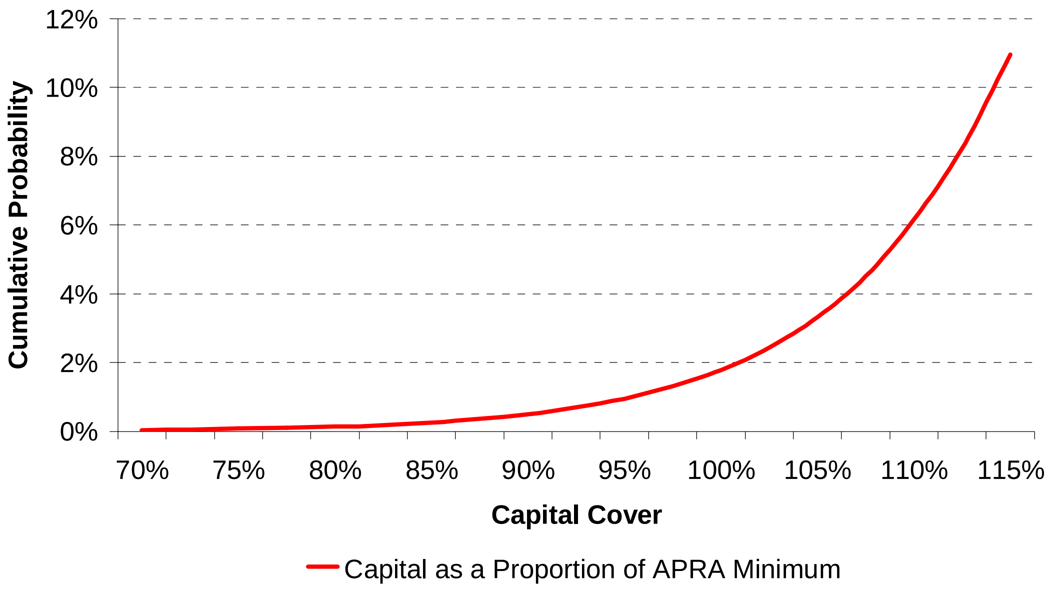
6. Does the Dependency Structure Make a Difference?

The Choice of Structure

The choice of dependency remains largely academic unless there is a clear impact on probability of failure. Below we use the worked example to highlight the impact that can arise through choice of dependency structure.

Figure 6.1 below shows the cumulative density function for capital cover (APRA Capital Base as a proportion of APRA Prescribed Minimum Capital Requirement) the worked example at the end of the year.

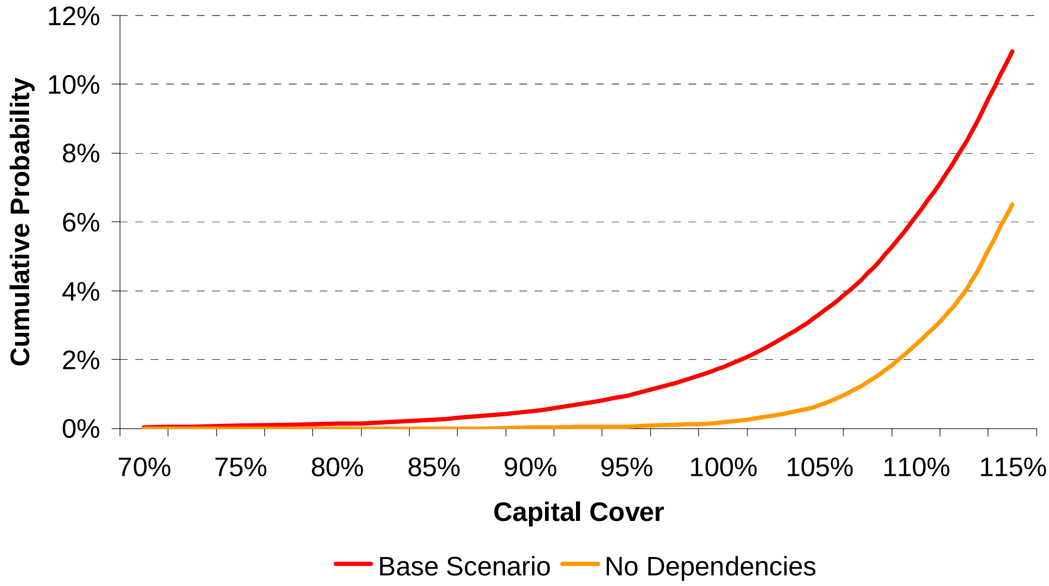
Figure 6.1 - Cumulative Capital Cover at Year End



The worked example gives the insurer's estimated probability of failure (defined as falling below 100% of APRA Minimum Capital) at year end at 1.8%.

Figure 6.2 shows the impact of removing the Gumbel dependency structure from the model. This is analogous to assuming the insurer's working losses are independent.

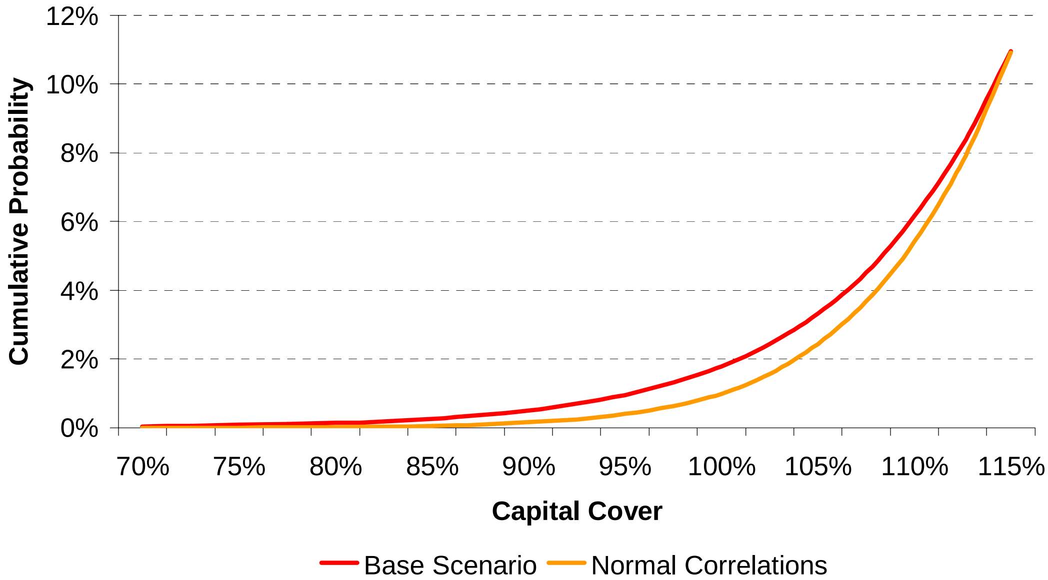
Figure 6.2 - Cumulative Capital Cover at Year End – No Dependencies



Were we to assume that the insurer's working losses were independent, this would reduce the estimated probability of failure to 0.2%, compared to 1.8% under the base scenario. So it is clear that the impact of some form of dependence can be a major driver in determining failure.

However, it does not necessarily show that choice of dependency structure is important. Figure 6.3 below shows the impact of choosing normal correlation coefficients which give rise to the same rank correlations as the chosen Gumbel parameters. This is as a proxy for parameterising the dependencies using linear correlations.

Figure 6.3 - Cumulative Capital Cover at Year End – Normal Correlations



By assuming normal correlations instead of a right tailed dependency structure the estimated probability of failure reduces to 1.0%, compared to 1.8% using the Gumbel copula. This means that the choice of dependency structure is a very important element of any capital model designed to analyse tail probabilities. Interestingly, the two curves converge at higher levels of projected capital cover.

DFA practitioners need to be confident that the dependencies modelled adequately measure the accumulation of risk. Choosing a structure based on normal correlations when a right tailed structure is more appropriate will lead to underestimation of the probability of failure. This can lead to insurers inadvertently accepting a greater level of risk than their appetite desires. Alternatively, insurers may need to rethink the level of risk they are prepared to accept.

Furthermore, there are immediate consequences for any insurer wishing to pursue the APRA Internal Model route. APRA's GGN 110.2 specifies that insurers need to "*… evaluate the interrelationship between… risks…*" as a key component of any internal model. As a result, choice of dependency structures should play a major role in the suitability of any APRA Internal Model.

The Importance of Dependence

Few benchmarks exist for the key dependency parameters (regardless of dependency structure chosen) and those that do exist do not always agree. Inevitably this means the DFA practitioner can legitimately justify a range of possible dependencies. Furthermore, the uncertainty is usually around the absolute level of the parameters, with the relativities not so uncertain. For example, the tail dependence between Liability and Professional Indemnity intuitively should be stronger than that between Liability and Motor, regardless of the absolute levels of tail dependence. Therefore, any estimation error for dependencies is likely to be across all or most links.

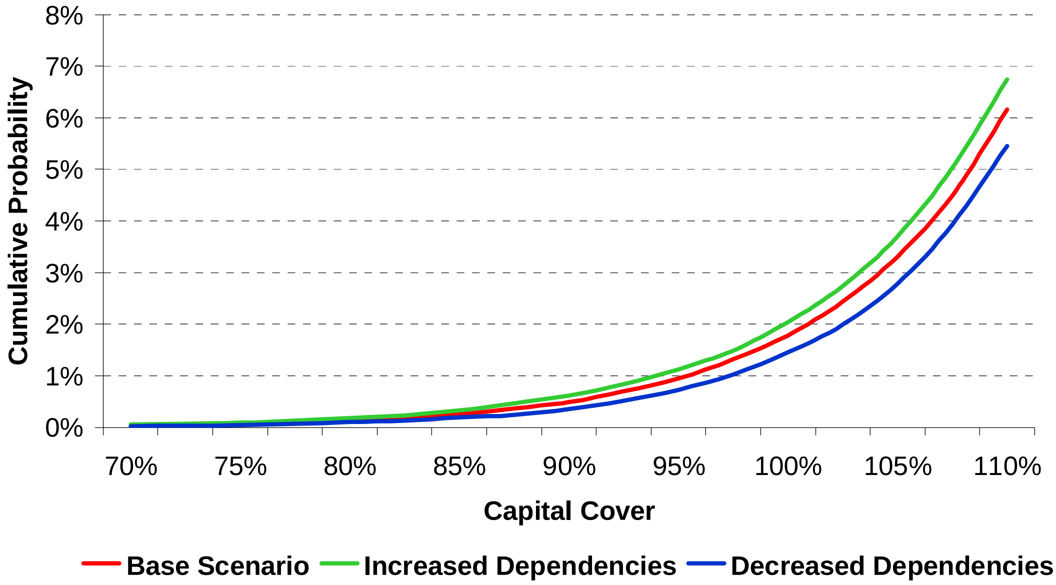
Table 6.1 below shows a possible range around the dependency assumptions. Whilst this is not intended to be a confidence interval in a statistical sense, it attempts to produce an indication of a reasonable range which the practitioner may choose as part of their model parameterisation.

Table 6.1 - Range of Possible Gumbel Parameters

Assumption	Selected Value	Low Value	High Value
Dependence			
CTP to Liability	Current	-0.1	+0.15
Casualty Classes	Current	-0.2	+0.20

Figure 6.4 below shows the capital cover where we select the low and high dependency parameters. The impact on probability of failure means that choice of dependency parameters is very important element of any DFA model.

Figure 6.4 - Estimation Error Due to Dependency Parameters



The range around the probability of failure due to choice of dependency structure is 1.5% to 2.0%.

This also leads to consequences for insurers developing an APRA Internal Model. GGN 110.2 recommends that stress testing and sensitivity analysis of assumptions within the capital measurement model take place. Given the level of judgement required to parameterise a dependency structure, a substantial level of stress testing should be expected not simply around the dependency parameters chosen, but also around the choice of dependency structure and relational structure where appropriate.

7. Conclusion

APRA has recognised that not all insurers' risk portfolios are the same. Via the introduction of the Internal Model, insurers are allowed to manage their own risk appetite outside of a prescriptive regime, but still subject to an absolute target. However, in doing so APRA requires insurers to understand their key sources of risk accumulation and drivers of failure. One of APRA's most important elements of any model testing process will be to understand the justification for the dependencies.

Outside of a use as a regulatory tool, many Australian insurers have implemented Dynamic Financial Analysis models as a means of maintaining a targeted risk management strategy. Often these strategies revolve around meeting a targeted probability of failure.

In estimating the probability of failure the DFA practitioner has a wide range of possible dependencies at their disposal. Whilst most practitioners are familiar with the concept of normal correlation, the choice should not be limited by familiarity. One possible alternative is the Gumbel dependency. The Gumbel is an extreme value copula, allowing the actuary to adequately model relationships in the right hand tail.

The choice of dependency structure plays an integral part in the estimation of an insurer's probability of failure. Assuming linear correlation where tail dependence is more appropriate can lead to substantial underestimation of the probability of failure. This in turn can lead to inappropriate risk strategies as insurers are lulled into believing they are less risky than they actually are.

Both insurers and APRA have a great deal at stake in the insurer's ability to estimate the accumulation of risk. Choosing an appropriate dependency structure plays a major part in ensuring risk exposures adequately match risk appetites.

8. References

APRA Prudential Standards GPS 110, GPS 210

APRA Guidance Notes GGN 110.1, GGN 110.2, GGN 110.3, GGN 110.4, GGN 110.5

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9. Appendices

Appendix A - Choosing a Suitable Dependency Structure

In extending beyond the traditional framework offered by Pearson correlations we are given a choice of many different copulas to utilise in modelling dependence. There are a number of considerations when choosing an appropriate structure for dependencies, including:

- The purpose of the stochastic model
- The extent to which other dependencies are already captured in the model
- The availability of data to parameterise the dependency structure.

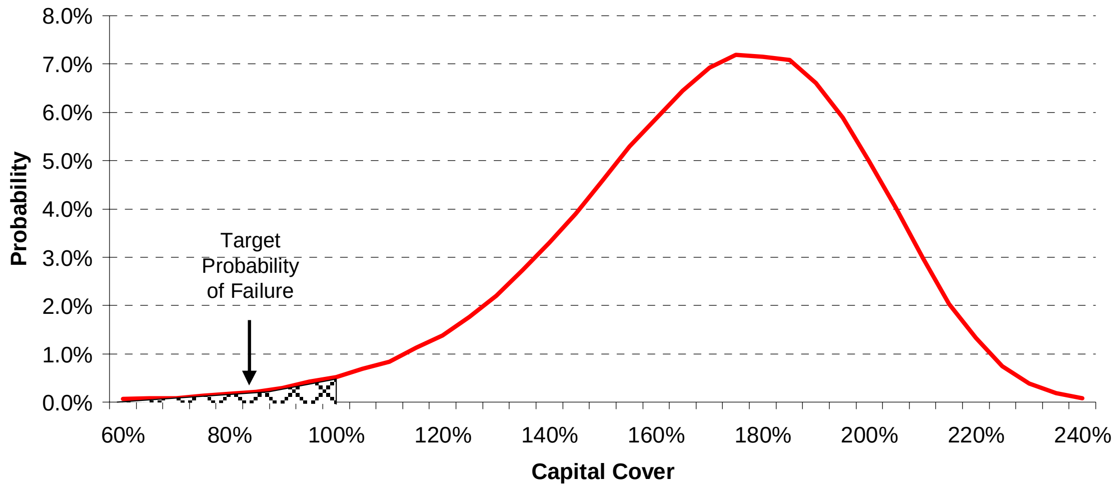
The Purpose of the Model

There are a wide range of possible uses for a stochastic model. Regardless of the purpose it is important to consider the elements of the results or the distributions that will most heavily influence the strategic decisions to be implemented. Needless to say, it is these aspects of the results where the dependencies should be modelled to be as realistic as possible. Two of the more common uses for stochastic models in Australia are to analyse tail behaviour and to determine a 75th percentile of sufficiency for technical liabilities. Considerations for dependencies for these two purposes are discussed below.

Modelling Tail Behaviour

Stochastic models are used to assist in understanding the likelihood of extreme events, allowing insurers to construct risk strategies to limit the chance of financial ruin or stress to an acceptable level. Many insurers benchmark an acceptable level of capital sufficiency as one offering a probability of ruin of 1 in 50 or better on a one year time horizon. Indeed APRA benchmark solvency at 99.5% or 1 in 200 or better on a one year time horizon as acceptable. For insurers, maintaining this benchmark can involve strategic choices in reinsurance, investment strategy and operational decisions.

Figure A.1 - Typical Capital Adequacy Target



Under such circumstances it is important to accurately model dependencies in the tail of the capital distribution, since the probability of ruin needs to be below 2.0% (and often significantly less than this) on a one year time horizon to meet capital sufficiency. Often this would suggest an extreme value copula, such as a Gumbel, which allows the practitioner to more accurately describe the relationship between variables in the extreme right hand tail of the distributions.

Modelling a 75th Percentile of Sufficiency

When determining a 75th percentile of sufficiency for outstanding claims reserves and premium liabilities reserves the actuary is less concerned with the extreme outcomes that are possible. Rather the actuary needs to ensure that the body of the distribution is accurately represented.

In determining a 75th percentile of sufficiency, the actuary can also have access to a substantial amount of information regarding the pair-wise relationships between classes, particularly for a mature portfolio of classes. These pair-wise relationships are clearly much easier to estimate at a 75th percentile than in the tail. Furthermore, the actuary has available a number of documented benchmarks for typical correlations between classes of business, for example the Tillinghast and Trowbridge reports.

Therefore, it appears sensible to adopt some form of correlation as a measure of dependence. Although normal correlation appears to be the default, rank correlation may be more appropriate.

Implicit versus Explicit Dependence

Identifying sources of dependence and modelling them explicitly is another method of modelling behaviour in the tail. There are many such sources of dependence that can be identified, isolated and modelled separately. These include:

- Catastrophe events impacting on multiple classes
- Reinsurance across multiple classes
- Consistent reserving philosophies, especially where the same actuary is responsible for reserving across multiple classes
- Superimposed inflation impacting on multiple classes
- Underwriting cycle
- Large losses
- Reinsurer failure
- Movement in economic conditions, in particular movement in the yield curve giving rise to changes in discount rates

One of the advantages of explicitly modelling certain sources of dependence is that there can be a direct impact between key causal events. For example, modelling catastrophe losses on an event basis, rather than a class basis makes sense if that gives rise to an event-based reinsurance recovery across multiple classes.

In certain circumstances enough of the dependencies can be explicitly modelled that no additional source of dependence need be incorporated into the model. As an example, using a stochastic model to determine a fair price for an aggregate excess of loss catastrophe reinsurance treaty may require explicit modelling of the underlying catastrophe events. These can be derived from previous analysis, such as that by RMS and can be isolated geographically to ensure that weather-based catastrophe events are, for all intents and purposes, independent. Hence, there is no need to construct further implicit dependencies in the model.

However, modelling explicit dependencies can introduce an added complication. In particular, it can be difficult to isolate the impact of certain dependencies. Explicitly modelling some but not all possible dependencies introduces a need to estimate the "remaining" dependencies. This can be extremely difficult to do in practice.

There are two reasons why this is often impractical, namely:

- It can be difficult to measurably identify the impact of the explicit dependence. For example, by modelling superimposed inflation across a number of classes as an explicit dependence requires the practitioner to estimate the dependencies excluding the impact of superimposed inflation. But superimposed inflation itself is extremely difficult to measure even for one class of business. Therefore, isolating and measuring the impact of superimposed inflation on tail dependence often becomes a matter of guesswork.
- Often two or more sources of explicit dependence are themselves dependent. For example, the insurer's view on the competitiveness of their own rates through the underwriting cycle is likely to be impacted by the actuary's current view of the adequacy of reserves. So to exclude the impact of the underwriting cycle and the reserve adequacy on the "remaining" dependencies, it is necessary to also take into account the impact of the two sources of explicit dependence themselves interacting.

This means that in order to model dependencies explicitly, the impact of the dependencies needs to be readily measurable and often independent of other sources of explicit dependence.

Available Data

If the insurer has enough representative history then it may be possible to construct an empirical dependency structure based on actual experience. However, there are a number of issues that should be recognised.

- Survivorship – The insurer has survived for a long enough period in order to be able to have enough representative history available to construct an empirical dependency structure. That the representative history does not include any events that threaten the insurer's solvency may itself lead to the history not being representative of the possible range of simulated dependencies

- Relevance – The nature of the insurer's portfolio, philosophy and risk tolerance may change over time. The impact of a key historical dependency may well be very different going forward
- Statistical Significance – It is rare that an insurer will have enough representative data to be able to adequately estimate an empirical dependency structure in a capital modelling framework
- Multi-dimensionality – As was noted earlier for Gumbels, not all pair-wise dependencies maintain their structure when extended to a multi-dimensional environment. Therefore, when constructing an empirical dependency structure in order to ensure that the structure is maintained, it is necessary to model all possible variables within the one empirical structure, rather than as a series of pair-wise dependencies. This can quickly complicate the model

Ultimately this means that for most capital modelling, particularly where the insurer is assessing tail behaviour, using empirical dependencies is often impractical or difficult to parameterise.

Appendix B - Multi-Dimensionality and Gumbel Dependencies

Partial Exchangeability

One of the issues with choosing a Gumbel tail dependency structure is the problem of partial exchangeability. That is, there are $n(n-1)/2$ unique pair-wise estimates, yet the Gumbel structure allows the practitioner to specify at most $n-1$ parameters. Hence it may not be possible to construct a Gumbel dependency structure where each best estimate pair-wise parameter holds.

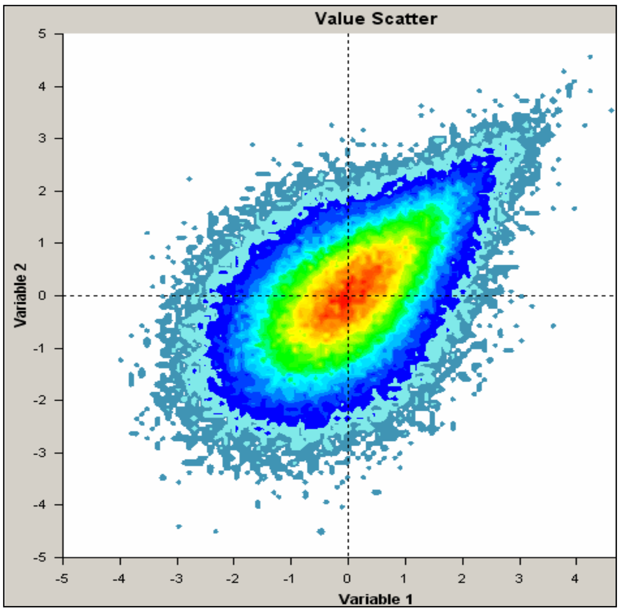
Ordering

To maintain the Gumbel "shape" when passing through the entire structure, the determination of the dependencies must be ordered in a specific way. The structure must be set up to order output so that the relationship between the two variables with the largest pair-wise parameter is simulated first, then the next largest and so on. Otherwise the only paired variables guaranteed to have a Gumbel structure will be the last two determined.

Negative Dependence

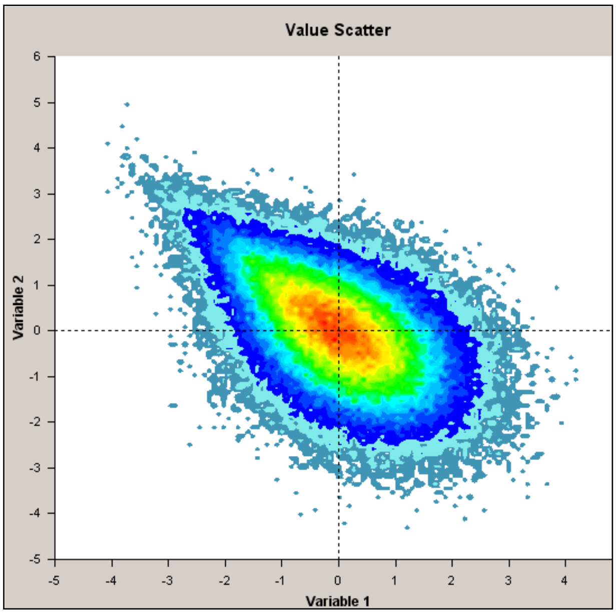
The Gumbel dependency structure is a right-tailed dependency. This means that the tail relationship is seen only in the upper right quadrant, as shown below in Figure B.1.

Figure B.1 - Normal Bivariates with Gumbel Dependence



However, problems can arise if you then model two classes, A and B, which are countercyclical. One method to assist with this is to negatively tie a Gumbel, so that the direction of the tail is reversed as shown in Figure B.2 below.

Figure B.2 - Normal Bivariates with Negatively Tied Gumbel



This means that if A performs very badly, then B is more likely to be performing exceptionally well. However, the counter is not necessarily true. One drawback is that if one of the classes (A say) is linked to a third class (C) then the other class (B) is automatically negatively tied to the third class (C). This may not be desirable.

More commonly, classes with such countercyclical properties are modelled using a different copula structure to the Gumbel, as right tail dependence is often less important for such classes.

Appendix C – Worked Example

The worked example consists of a model of eight classes of business being:

- Liability
- Workers' Compensation
- Compulsory Third Party
- Professional Indemnity
- Commercial Property
- Commercial Motor
- Home & Contents
- Domestic Motor.

Appendix C.1 shows a table of historical financial year industry loss ratios for 1993 to 2001 for the eight classes.

Appendix C.2 shows the assumptions for the DFA model underpinning the worked example.

Appendix C.1

Net Loss Ratios

Liability	Workers Comp	CTP	Professional Indemnity	Commercial Property	Commercial Motor	Domestic Property	Domestic Motor
67%	122%	82%	100%	70%	79%	64%	82%
71%	118%	91%	97%	43%	74%	65%	86%
67%	118%	144%	81%	45%	82%	64%	87%
80%	117%	141%	64%	47%	84%	60%	89%
85%	124%	110%	89%	53%	81%	64%	86%
101%	136%	103%	93%	52%	84%	55%	83%
140%	122%	87%	114%	62%	87%	63%	85%
126%	133%	92%	106%	65%	68%	60%	91%
125%	98%	87%	126%	60%	78%	56%	79%

Log of Net Loss Ratios

Liability	Workers Comp	CTP	Professional Indemnity	Commercial Property	Commercial Motor	Domestic Property	Domestic Motor
-0.404	0.202	-0.204	0.000	-0.358	-0.230	-0.451	-0.204
-0.344	0.168	-0.099	-0.034	-0.836	-0.306	-0.423	-0.150
-0.405	0.164	0.362	-0.215	-0.791	-0.200	-0.440	-0.143
-0.223	0.156	0.346	-0.441	-0.754	-0.175	-0.511	-0.111
-0.160	0.214	0.094	-0.122	-0.639	-0.216	-0.451	-0.148
0.014	0.306	0.025	-0.071	-0.653	-0.171	-0.599	-0.186
0.337	0.202	-0.136	0.128	-0.486	-0.138	-0.466	-0.168
0.231	0.282	-0.080	0.056	-0.431	-0.387	-0.510	-0.098
0.225	-0.018	-0.138	0.230	-0.509	-0.253	-0.587	-0.235

Mean and Std Dev

Liability	Workers Comp	CTP	Professional Indemnity	Commercial Property	Commercial Motor	Domestic Property	Domestic Motor
-0.0809	0.1863	0.0188	-0.0521	-0.6065	-0.2305	-0.4932	-0.1603
0.2914	0.0923	0.2099	0.1968	0.1690	0.0768	0.0640	0.0434

Normalised Log of Net Loss Ratios

Liability	Workers Comp	CTP	Prof Indemnity	Comm Property	Comm Motor	Dom Property	Dom Motor
-1.1077	0.1745	-1.0629	0.2632	1.4679	0.0105	0.6576	-1.0018
-0.9024	-0.1975	-0.5598	0.0940	-1.3588	-0.9776	1.0944	0.2278
-1.1118	-0.2388	1.6325	-0.8302	-1.0926	0.3996	0.8353	0.4036
-0.4890	-0.3261	1.5607	-1.9758	-0.8747	0.7260	-0.2763	1.1363
-0.2700	0.2959	0.3583	-0.3538	-0.1952	0.1909	0.6543	0.2832
0.3270	1.2991	0.0285	-0.0964	-0.2773	0.7789	-1.6559	-0.5825
1.4343	0.1724	-0.7378	0.9154	0.7131	1.2063	0.4303	-0.1674
1.0702	1.0329	-0.4708	0.5508	1.0416	-2.0431	-0.2669	1.4313
1.0494	-2.2125	-0.7488	1.4328	0.5760	-0.2915	-1.4728	-1.7304

Appendix C.2

Class of Business Assumptions

	Liability	Workers Comp	CTP	Professional Indemnity	Commercial Property	Commercial Motor	Home & Contents	Domestic Motor
Net Written Premium	10,297	8,262	21,876	6,194	17,273	11,297	24,558	37,523
Premium Earned	50%	50%	50%	50%	50%	50%	50%	50%
Disc Net Loss Ratio Mean	70%	65%	85%	65%	65%	70%	65%	60%
Disc Net Loss Ratio Std Dev	16%	12%	13%	16%	9%	8%	6%	5%
Payment Pattern								
Year 1	5%	25%	25%	5%	75%	80%	85%	85%
Year 2	15%	15%	15%	15%	15%	12%	10%	10%
Year 3	18%	12%	12%	18%	7%	5%	3%	3%
Year 4	12%	11%	11%	12%	2%	2%	1%	1%
Year 5	10%	10%	10%	10%	1%	1%	1%	1%
Year 6	10%	8%	8%	10%	0%	0%	0%	0%
Year 7	8%	6%	6%	8%	0%	0%	0%	0%
Year 8	7%	5%	5%	7%	0%	0%	0%	0%
Year 9	6%	4%	4%	6%	0%	0%	0%	0%
Year 10	9%	4%	4%	9%	0%	0%	0%	0%
Incurred Loss in Year	30%	50%	45%	30%	85%	88%	90%	85%

Appendix C.2

Company Assumptions	
Opening Capital	75,000
Expenses (% of NWP)	30%
Investment Return	5%
Discount Rate	5%
Maximum Event Retention	5,000
Average Inv Capital Charge	2%

Catastrophe Assumptions	
Frequency	1.5
Net Retention	1,000